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RATE OF CHANGE OF GRADE PER STATION

By Clarence J. Brownell, A. M. ASCE

HIGHWAY DIVISION

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PAPERS

RATE OF CHANGE OF GRADE PER STATION

By Clarence J. Brownell,[1] A. M. ASCE

Synopsis

Highway engineers generally describe a vertical curve by stating its length and the rates of the adjacent tangent grades. Although this definitely establishes the curve it does not give a comprehensive idea of its sharpness. A curve in horizontal alinement is described by its angle of intersection, radius or degree of curve, length and tangent distances, and the bearings of the adjacent tangents. These data, especially the radius, convey a very definite picture of the sharpness of the circular curve. A vertical curve is part of a parabola, computed and laid out using horizontal and vertical measurements. Being a parabola, a vertical curve has no constant radius or degree of curve, which probably accounts for the lack of such a factor in its description.

The purpose of this paper is to urge the adoption of a method of describing vertical curves which will give an immediate impression of the sharpness of such a curve. The method explained herein does that very thing, and further, it provides a convenient tool for the solution of problems in vertical alinement that can otherwise be determined only by trial and error. In the coming era of divided highways and on-and-off ramps, this tool will be found most convenient.

General

The method involves the use of a factor R which can be described as the "rate of change of grade per station." In a parabolic curve, R is the percentage change in grade of the tangent to the curve in one station of distance. For instance, if the grade G_1 of the tangent is 0 at one station and $G_2 = +1\%$ at the next station, the rate of change of grade per station is $+1\%$. If the grade is $G_1 = -5\%$ at one station and $G_2 = -7\%$ at the next station, the rate of change of grade per station is $\frac{G_2 - G_1}{1} = -2\%$. Positive rates of

Note.—Written comments are invited for publication; the last discussion should be submitted by September 1, 1952.

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change curve upward; negative rates of change curve downward. Vertical curves with the same rate of change of grade per station are parts of identical parabolas, regardless of their lengths or the grades of their adjacent tangents.

In Fig. 1 let: H be the difference in elevation between the beginning and the end of a vertical curve; L be the distance in stations between two points; and V be the tangent offset between the two tangents of a vertical curve. For the purpose of illustration, let $R = +1\%$ in Fig. 1(*a*) and let the grade at station 10 be $G_{10} = +1\%$. Since R is the percentage change in grade of the tangent to the curve in one station, the computed values for G for each station

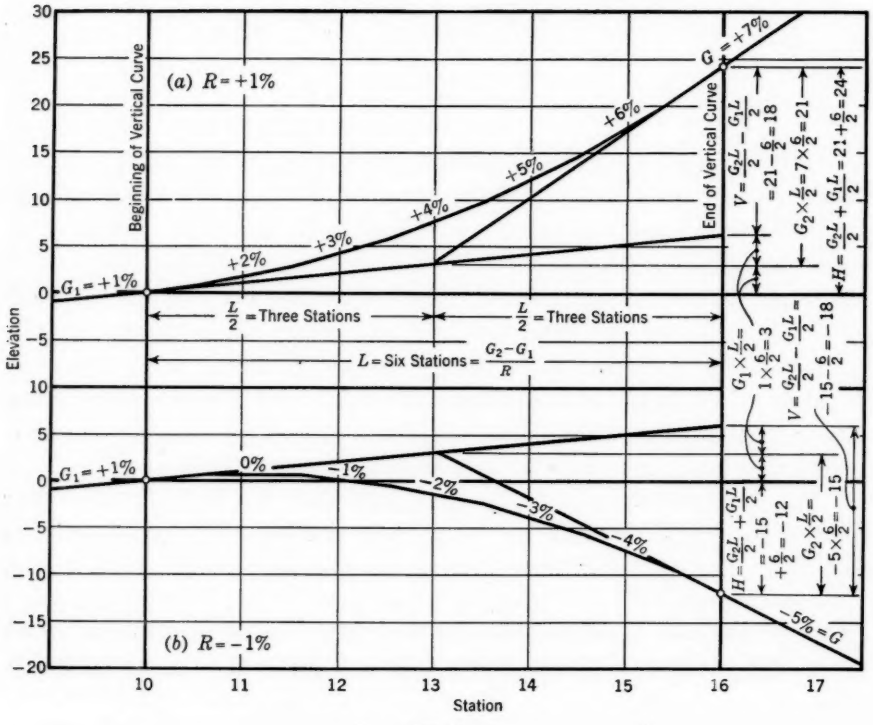


FIG. 1

are as shown directly on the curve in Fig. 1(*a*). For purposes of clarity and explanation, the curve has been drawn as a series of tangents, each one station in length. The tangents intersect at the + 50 points.

Now consider Fig. 1(*b*). Using the same grade of + 1% at station 10, but using a rate of change of grade per station of − 1%, the grade G at each station is shown along the curve. From inspection of the two curves in Fig. 1, and from the definition of R, it is apparent that the algebraic difference of the grades is equal to the length of the curve, in stations, multiplied by the rate of change of grade per station, or

$$G_2 - G_1 = L\,R \qquad (1)$$

in which G_1 is the grade at the first point on the curve; G_2 is the grade at the second point on the curve; L is the distance in stations between the two points; and R is the rate of change of grade per station. Symbols G_1 and G_2 denote the grades at the beginning and the end of the curve, or they may be the grades at any points on the curve, as long as L is the distance, in stations, between them. This distance L may be in stations of 100 ft or of any convenient length. It is suggested that "rate of change of grade per station" be abbreviated on plans as "R/C" or "Rc" in order to avoid confusing it with the abbreviation for the radius of a circular curve. The letter symbol R will be used in formulas. The tangent offset V (Fig. 1) is composed of two parts, $G_2 \frac{L}{2}$ and $G_1 \frac{L}{2}$. Keeping the direction of these values in mind, the equation for V can be written as

$$V = \frac{L}{2}(G_2 - G_1) \quad \ldots\ldots (2)$$

Consider the difference in elevation between any two points, designated H in Fig. 1. In order to maintain the established sign notation, H must be considered as positive when measured upward and negative when measured downward. Again by inspection, watching signs, the equation for H can be written as

$$H = \frac{L}{2}(G_2 + G_1) \quad \ldots\ldots (3)$$

Eqs. 1, 2, and 3 express the three basic principles. From them, and combinations of them, any problem in vertical curves can be solved. The reader is urged to familiarize himself thoroughly with the graphical presentation. The formulas will then become second nature, and there will be no need to memorize them. Consider the formulas merely as a concise and convenient method of expressing a thought—as engineering "shorthand."

By combining Eqs. 3 and 1 the expression,

$$H = G_1 L + \frac{R}{2} L^2 \quad \ldots\ldots (4a)$$

can be written. This is the equation of a parabola, symmetrical about a vertical axis with its origin on the curve at a point where the tangent to the curve has a grade of G_1. Such an equation in the usual x-coordinates and y-coordinates would be written

$$y = \frac{a}{p} x + \frac{1}{2p} x^2 \quad \ldots\ldots (4b)$$

in which a is the horizontal distance from the vertex to the origin of coordinates and p is the distance from the vertex to the focus of the parabola. It is apparent by comparison that $R = \frac{1}{p}$ and $G_1 = \frac{a}{p}$, from which

$$a = \frac{G_1}{R} \quad \ldots\ldots (5)$$

The grade at any point on the curve may be described mathematically as the first derivative of the equation of the curve:

$$\frac{dH}{dL} = \frac{d}{dL}\left(G_1 L + \frac{R}{2} L^2\right) = G_1 + R L = G_2 \ldots\ldots\ldots\ldots (6a)$$

which is in agreement with Eq. 1. The rate of change of grade per station may be described mathematically as the second derivative of the equation of the curve:

$$\frac{d^2H}{dL^2} = \frac{d^2}{dL^2}\left(G_1 L + \frac{R}{2} L^2\right) = R \ldots\ldots\ldots\ldots\ldots\ldots (6b)$$

The same principle holds for parabolic curves of other than the second power; but the second derivative of such an equation is not a constant.

The statement of a + 10% grade, immediately conjures up a very definite impression to any highway engineer. Likewise, a − 12% grade gives another immediate impression; and if a vertical curve one station long were to be placed between them it would be stated, without hesitation, to be too short. Mentally, a rate of change of grade of − 22% per station was pictured without being aware of why or how it was done. Knowingly describing vertical curves by the method of rate of change will give an immediate appreciation of their sharpness. Vertical curves occur on highways far more frequently than do horizontal curves, and they can be just as dangerous. Therefore, they should be described and analyzed every bit as completely. The factor, "rate of change," will provide that description and analysis.

Typical Examples

To illustrate the use of "rate of change," the following typical problems are presented, with a clue for their solution.

Example 1.—A common occurrence is the need for the elevation at a few intermediate points along a vertical curve, such as at the abutments and piers of a bridge. These can most readily be determined by tabulation, as in Table 1. All the necessary steps have been shown. With familiarity, the computations would probably be made on scratch paper and only Cols. (1), (2), (5), and (8) would be tabulated. The computations close on the EVC thereby making a definite check on all values. Instead of a tabulation of tangent elevations and corrections, which have no further value, there is a tabulation of the grade at each point, which is of inestimable value.

Example 2.—It is frequently desirable to tabulate the elevations at each station along a vertical curve. For this purpose it is convenient to consider the curve as made up of a series of chords between stations. The grade of such a chord is equal to the grade of the tangent to the curve midway between stations. For proof of this statement consider that the grade of the chord (Fig. 2) is equal to the difference in elevation (H) divided by the distance (L) between points. From a combination of Eqs. 1 and 3, $H = G_1 L + R\frac{L^2}{2}$; therefore,

$$\frac{H}{L} = G_1 + R\frac{L}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

By reference to Eq. 1 the grade at $\frac{L}{2}$ can be written as $G_1 + R\frac{L}{2}$. Hence $\frac{H}{L}$ is the grade of the tangent at $\frac{L}{2}$.

TABLE 1.—COMPUTATION OF INTERMEDIATE ELEVATIONS ON A VERTICAL CURVE (Rate of Change R equals 0.4%)

Procedure:

Col. (3), the interval between stations in Col. (1).
Col. (4), the difference between grades, as given by Eq. 1; is found by multiplying Col. (3) by 0.4%.
Col. (5), $G_2 = G_1 + R\,L$, from Eq. 1; thus, at Sta. 12+50, $G_2 = -0.22$ from Col. (5), plus $+0.12$ from Col. (4), equals -0.10.
Col. (6), the average percentage grade, is the average of adjacent values in Col. (5).
Col. (7), the difference of elevation, in feet, computed by Eq. 3, is the product of Cols. (3) and (6).
Col. (8), elevation E_2 ($= E_1 + H$), in feet, is computed by adding the proper value from Col. (7) to the next preceding elevation (E_1) in Col. (8); thus, at Sta. 12+50, $E_2 = 98.548$, from Col. (8), plus -0.048 from Col. (7), equals 98.500.

Station	Description	Length, L	$G_2 - G_1$ (Eq. 1)	Grade, G_2	$\frac{G_2 + G_1}{2}$	H, in feet	Elevation, E_2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
16+00	End of vertical curve (EVC)			+1.30			100.600
		2.80	+1.12		+0.74	+2.072	
13+20	Abutment No. 4			+0.18			98.528
		0.30	+0.12		+0.12	+0.036	
12+90	Pier No. 3			+0.06			98.492
		0.40	+0.16		−0.02	−0.008	
12+50	Pier No. 2			−0.10			98.500
		0.30	+0.12		−0.16	−0.048	
12+20	Abutment No. 1			−0.22			98.548
		2.20	+0.88		−0.66	−1.452	
10+00	Beginning of vertical curve (BVC)			−1.10			100.000

A sample tabulation with explanatory headings and side notes is shown below. In Table 2, the grade at each + 50-point is obtained by adding the rate of change to the preceding + 50-point; and the elevation at ecah station is found by adding the grade at the midpoint to elevation at the preceding

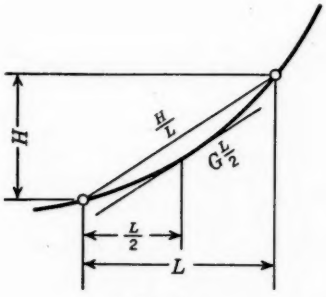


FIG. 2

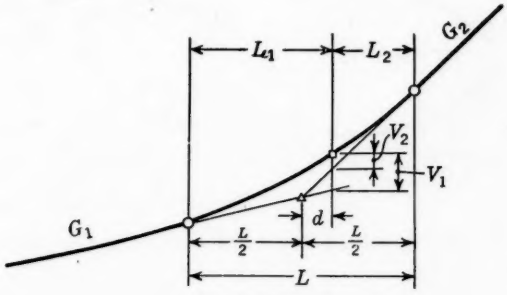


FIG. 3

station. If the elevations are desired at the + 50-points, repeat the process by writing the grades at the stations and the elevations at the + 50-points. The process can be repeated to obtain elevations at any intervals desired.

When completed there is a tabulation of grades at each point, rather than tangent elevations and curve corrections which have no meaning. Note that the computations close on the EVC, providing a positive check on the entire tabulation.

Example 3.—A problem that frequently arises is to find the curve that will pass through a known point and meet a pair of given tangents. There is only one curve that will meet this condition. Consider the curve in two parts (see Fig. 3). Since it is to be the same continuous curve, the rate of change will be the same for each part. From Eqs. 1 and 2:

$$R = \frac{2\,V}{L^2} \ldots\ldots (8)$$

For the first part of the curve, $R = \frac{2\,V_1}{L^2_1}$, in which V_1 is the vertical offset from the first tangent and L_1 is the distance from the BVC to the point. For the second part of the curve, $R = \frac{2\,V_2}{L^2_2}$, in which V_2 is the vertical offset from the second tangent and L_2 is the distance from the point to the EVC. If the distance from the vertical point of intersection, VPI, to the point is designated d, Fig. 3, and if the total length of the curve is L, then $L_1 = \frac{L}{2} + d$ and $L_2 = \frac{L}{2} - d$, noting the proper sign for d. Since, then, $\frac{2\,V_1}{\left(\frac{L}{2} + d\right)^2} = \frac{2\,V_2}{\left(\frac{L}{2} - d\right)^2}$, the final step is to evaluate V_1, V_2, d, and solve for L.

TABLE 2.—COMPUTATION OF ELEVATIONS, EACH STATION ALONG A VERTICAL CURVE[a] ($R = +0.5\%$, Example 2)

Station[b] (1)	Grade, G_2 (5)	Elevation, E_2 (8)	Procedure[a] (2)
6+70	+1.25	100.5625	99.81+(1.075×0.70)
			+1.075+(0.5×0.35)
6+35	+1.075		−0.65+(0.5×0.85)
6+00		99.81	99.16+(+0.65×1.00)
5+50	+0.65		+0.15+(0.5×1.00)
5+00		99.16	99.01+(+0.15×1.00)
4+50	+0.15		−0.35+(0.5×1.00)
4+00		99.01	99.36+(−0.35×1.00)
3+50	−0.35		−0.80+(0.5×0.90)
3+00		99.36	100.00+(−0.80×0.80)
2+60	−0.80		−1.00+(0.5×0.40)
2+20	−1.00	100.00	Beginning vertical curve[b]

[a] The column numbers and the procedure are the same as in Table 1 and Example 1. [b] In the notes, the beginning of the vertical curve is designated BVC and the end of the vertical curve (Sta. 6+70, is designated EVC.

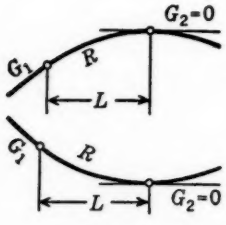


FIG. 4

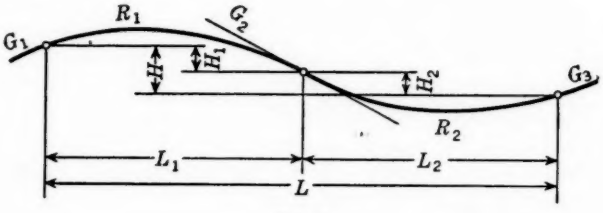


FIG. 5

Obviously, this will be the point at which the grade of the tangent to the curve is 0%. Eq. 1, solved for L, with $G_2 = 0$, will give the distance from the BVC to the lowest or highest point. The elevation of this point (see Fig. 4) can be found from Eq. 3.

Example 5.—The determination of the vertical alinement for an interchange ramp amounts to evaluating the data for a pair of reverse curves between the two roads. The complete data (see Fig. 5) will consist of the following:

Description	Symbol
Grade of the approaching tangent	G_1
Grade of the common tangent	G_2
Grade of the receding tangent	G_3
Length of the first curve	L_1
Length of the second curve	L_2
Vertical change in the first curve	H_1
Vertical change in the second curve	H_2
Rate of change of the first curve	R_1
Rate of change of the second curve	R_2
Over-all length $(L_1 + L_2)$	L
Over-all difference in elevation $(H_1 + H_2)$	H

The solution can be divided into various cases dependent upon the known and assumed data.

Case I. Values of G_1, G_3, and L Are Known.—Assume values for R_1 and R_2. As a guide for these assumptions use the charts presented subsequently as Figs. 7, 8, and 9. The fundamental formula, Eq. 1, can be written:

$$L_1 = \frac{G_2 - G_1}{R_1} \qquad (9a)$$

and

$$L_2 = \frac{G_3 - G_2}{R_2} \qquad (9b)$$

By adding Eqs. 9,

$$L = \frac{G_2 - G_1}{R_1} + \frac{G_3 - G_2}{R_2} \qquad (10a)$$

which can be reduced to

$$G_2 = \frac{R_1 (R_2 L - G_3) + R_2 G_1}{R_2 - R_1} \qquad (10b)$$

With this value of G_2, the values of L_1 and L_2 can be found from Eqs. 9. The values of H_1 and H_2 can be found from Eqs. 3, written as

$$H_1 = \frac{L_1}{2} (G_2 + G_1) \qquad (11a)$$

and

$$H_2 = \frac{L_2}{2} (G_3 + G_2) \qquad (11b)$$

Also

$$H = H_1 + H_2 \qquad (12)$$

Case II. Values of G_1, G_3, and H Are Known.—Assume values for R_1 and R_2 using the charts for sight distance (Figs. 7, 8, and 9) as a guide. The fundamental formulas, Eqs. 1 and 3 can be combined and written:

$$H_1 = \frac{G^2{}_2 - G^2{}_1}{2\,R_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13a)$$

and

$$H_2 = \frac{G^2{}_3 - G^2{}_2}{2\,R_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13b)$$

By adding these two equations,

$$H = \frac{G^2{}_2 - G^2{}_1}{2\,R_1} + \frac{G^2{}_3 - G^2{}_2}{2\,R_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14a)$$

which can be reduced to

$$G_2 = \sqrt{\frac{R_1\,(2\,R_2\,H - G^2{}_3) + R_2\,G^2{}_1}{R_2 - R_1}} \quad \ldots\ldots\ldots\ldots (14b)$$

Having determined the value of G_2, proceed as in Case I to find the other data.

Case III. Values of G_1, L, and H Are Known.—Assume values for R_1 and R_2 using the charts for sight distance (Figs. 7, 8, and 9) as a guide. Equating the expressions for G_2 as found in Cases I and II, and reducing,

$$G_3 = G_1 + R_1\,L + \sqrt{(R_1 - R_2)(R_1\,L^2 + 2\,G_1\,L - 2\,H)} \quad \ldots\ldots (15)$$

Having the value of G_3, proceed as in Case I to find G_2 and the other data.

Case IV. Values of G_3, L, and H Are Known.—Assume Values for R_1 and R_2 using the charts for sight distance (Figs. 7, 8, and 9) as a guide. Equating the expressions for G_2 as found in Cases I and II, and reducing,

$$G_1 = G_3 - R_2\,L + \sqrt{(R_2 - R_1)(R_2\,L^2 - 2\,L\,G_3 + 2\,H)} \quad \ldots\ldots (16)$$

Having determined the value of G_1 proceed as in Case I to find G_2 and the other data.

Case V. Values of G_1, G_3, L, and H Are Known, and the Value of R_1 Is Known or Assumed.—Equating the expressions for G_2 as found in Cases I and II, and reducing,

$$R_2 = \frac{2\,R_1\,(L\,G_3 - H) - (G_3 - G_1)^2}{L\,(R_2\,L - 2\,G_3) + 2\,H} \quad \ldots\ldots\ldots\ldots (17)$$

Having found the value of R_2, proceed as in Case I to find G_2 and the other data.

Case VI. Values of G_1, G_3, L, and H Are Known, and the Value of R_2 Is Known or Assumed.—Equating the expressions for G_2 as found in Cases I and II, and reducing,

$$R_1 = \frac{2\,R_2\,(H - L\,G_1) - (G_3 - G_1)^2}{L\,(R_2\,L - 2\,G_3) + 2\,H} \quad \ldots\ldots\ldots\ldots (18)$$

Having found the value of R_1 proceed as in Case I to find G_2 and the other data.

Case VII. Values of G_1, G_3, L, and H Are Known, and the Values of R_1 and R_2 Are Unknown, but the Greatest Possible Sight Distances Are Desired.—Assume values for R_1 and R_2 using the charts for sight distance (Figs. 7, 8, and 9) as a guide. Let their ratio $\frac{R_1}{R_2} = C$; then $R_1 = C\,R_2$. Equating the expressions for G_2 as found in Cases I and II but replacing R_1 with its equivalent $C\,R_2$, and reducing,

$$R_2 = \frac{1}{C\,L^2}\{\sqrt{[L\,(G_1 - C\,G_3) + H\,(C-1)]^2 - C\,L^2\,(G_3 - G_1)^2} - [L\,(G_1 - C\,G_3) + H\,(C-1)]\} \quad (19a)$$

Find the value of R_1 from its ratio $R_1 = C\,R_2$ and determine the remaining data by the method of Case I. It may be more convenient to express the ratio as $\frac{R_2}{R_1} = K$; then $R_2 = K\,R_1$ and

$$R_1 = \frac{1}{K\,L^2}\{\sqrt{[L\,(K\,G_1 - G_3) + H\,(1-K)]^2 - K\,L^2\,(G_3 - G_1)^2} - [L\,(K\,G_1 - G_3) + \mathrm{H}\,(1-K)]\} \quad (19b)$$

Find the value of R_2 from its ratio $R_2 = K\,R_1$ and determine the remaining data by the method of Case I.

In using these equations it is absolutely necessary that sign notations be followed strictly:

$+ R$ is used for rising curves,
$- R$ is use for curves leading downward,
$+ H$ indicates that the second point is higher than the first,
$- H$ indicates that the second point is lower than the first,
$+ G$ indicates grades that slope upward, and
$- G$ indicates grades that slope downward.

The problem has been illustrated using R_1 as negative (summit) and R_2 as positive (sag). The same equations will hold if the problem is set up in the reverse order.

The vertical curve of a desired rate of change of grade per station can be passed through two known points by using a combination of Eqs. 1 and 3 in the form $G_1 = \frac{H}{L} - \frac{L\,R}{2}$ and $G_2 = \frac{H}{L} + \frac{L\,R}{2}$. The vertical curve that will pass through three points can be determined by using the foregoing expressions and considering the curve in two parts, with a common R and a common tangent at the intermediate point. The point on a vertical curve at which its tangent produced will pass through a given point outside the curve can be readily found using the rate of change of grade per station.

Sight Distance Over a Summit

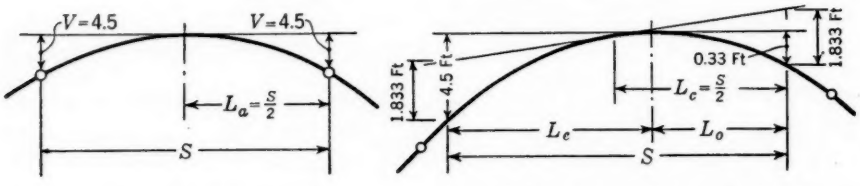
The "passing sight distance over a summit" has been defined as the distance between points 4.5 ft vertically above the road surface, which are visible one

to the other. "Nonpassing sight distance" is defined as the distance at which an observer, with his eye 4.5 ft above the surface can see an object 4 in. above the surface of the road.[2] The sight distance, S, over a summit can be readily expressed in terms of "rate of change R."

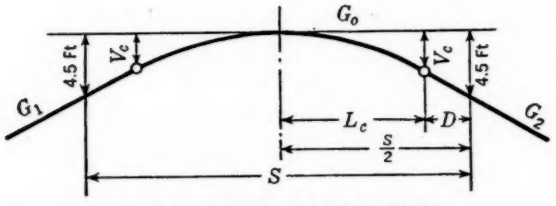
Passing Sight Distance.—When the length of the vertical curve is equal to or greater than the sight distance, Eq. 8 can be solved for L_a. Referring to Fig. 6(*a*): $L_a = \frac{S}{2} = \sqrt{\frac{2(-4.5)}{R}}$; and, $S = \frac{6\sqrt{-1}}{\sqrt{R}}$. Since R is negative, the sight distance S will be equal to

$$S = \frac{6}{\sqrt{R}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

When the sight distance S is greater than the length of the vertical curve, it will be made up of two parts (see Fig. 6(*b*)), the length of the curve and parts



(*a*) $S<L$, PASSING SIGHT DISTANCE (*c*) $S<L$, NONPASSING SIGHT DISTANCE



(*b*) $S<L$, PASSING SIGHT DISTANCE

FIG. 6

of the tangents at each end. It can be expressed as

$$S = 2(L_c + D) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

From Eq. 1, L_c can be expressed as

$$L_c = \frac{G_2 - G_c}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22a)$$

[2] "A Policy on Sight Distance for Highways," Am. Assn. of State Highway Officials, 1940, p. 17.

The distance D along the forward tangent (Fig. 6(b)) can be expressed as

$$D = \frac{4.5 - V_c}{G_2 - G_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22b)$$

Substituting Eqs. 22 in Eq. 21,

$$S = 2\left(\frac{G_2 - G_c}{R} + \frac{4.5 - V_c}{G_2 - G_c}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

In Eq. 23, substituting,

$$V_c = \frac{(G_2 - G_c)^2}{2R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

the sight distance becomes

$$S = 2\left(\frac{G_2 - G_c}{2R} + \frac{4.5}{G_2 - G_c}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (25a)$$

Since G_c is the grade at the center of the curve, it can be expressed as $G_c = \frac{G_2 + G_1}{2}$, and $(G_2 - GF_c)$ becomes $\frac{G_2 - G_1}{2}$. Therefore, Eq. 25a becomes

$$S = \frac{18}{G_2 - G_1} + \frac{G_2 - G_1}{2R} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25b)$$

When $S = L$ (Fig. 6(c)), Eq. 1 can be written $R = \frac{G_2 - G_1}{S}$ and Eq. 25b becomes

$$S = \frac{18}{G_2 - G_1} + \frac{G_2 - G_1}{\frac{2\,(G_2 - G_1)}{S}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

and

$$S = \frac{36}{G_2 - G_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

Eq. 27 must be equal to Eq. 20; therefore, if $G_2 - G_1$ in Eq. 27 is replaced by $R\,S$ then $S = \frac{36}{R\,S}$, and again $S = \frac{6}{\sqrt{R}}$.

Nonpassing Sight Distance.—For nonpassing sight distance, V has two different values—4.5 ft and 0.33 ft. By Eq. 8, $L = \sqrt{\frac{2\,V}{R}}$, $L_o = \sqrt{\frac{2(0.33)}{R}} = \frac{0.667}{R}$, and $L_e = \sqrt{\frac{2(4.5)}{R}} = \sqrt{\frac{9}{R}}$. Therefore (compare Eq. 20):

$$S = L_o + L_e = \frac{0.8165 + 3}{\sqrt{R}} = \frac{3.8165}{\sqrt{R}} \quad \ldots\ldots\ldots\ldots\ldots (28)$$

In order to simplify the data so that the second part of the problem will be similar to the one already discussed, it is convenient to find the tangent

offset for one half the total sight distance. This can be done easily by working the foregoing data backward. Since $S = \sqrt{\frac{14.662}{R}}$, $\frac{S}{2}$ (or L_c) $= \sqrt{\frac{3.666}{R}}$; and since $L_c = \sqrt{\frac{2\,V_c}{R}}$, the tangent offset $V_c = \frac{3.666}{2} = 1.833$.

By the same reasoning as before, Eq. 25*b* becomes, for nonpassing sight distance,

$$S = \frac{7.331}{G_2 - G_1} + \frac{G_2 - G_1}{2\,R} \quad \ldots\ldots\ldots\ldots\ldots\ldots (29a)$$

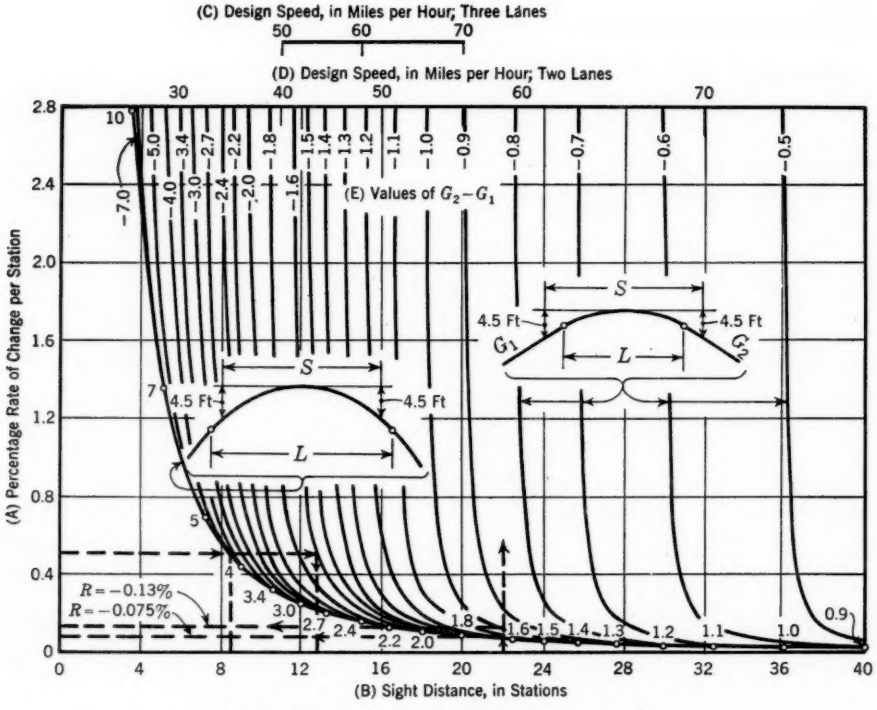


FIG. 7.—PASSING SIGHT DISTANCE OVER A SUMMIT

and Eq. 28 becomes

$$S = \frac{14.662}{G_2 - G_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29b)$$

The results of Eqs. 20 and 25*b* for usable values of R and $G_2 - G_1$ are given in Table 3(*a*) and are represented graphically by Fig. 7. The results of Eq. 27 are given in Table 4(*a*) and are indicated on Fig. 7 by small circles at the junction of the curves. The results of Eqs. 28 and 29*a* are given in Table 3(*b*) and are represented graphically by Fig. 8. The results of Eq. 29*b* are given in Table 4(*b*) and are indicated on Fig. 8 by small circles at the junction of the

curves. (Scale-A values are negative when applied to Fig. 8 and positive when applied to Fig. 9.)

Values of S are readable from the charts to the nearest 5 ft and values of R to the nearest 0.01% per station. Straight-line interpolation between the values given in Table 3(a) will be slightly greater than calculated values. For example at a value of $R = -1.0\%$ per station and $G_2 - G_1 = -0.55\%$, the interpolated value of S will be 27 ft greater than the calculated value. For a

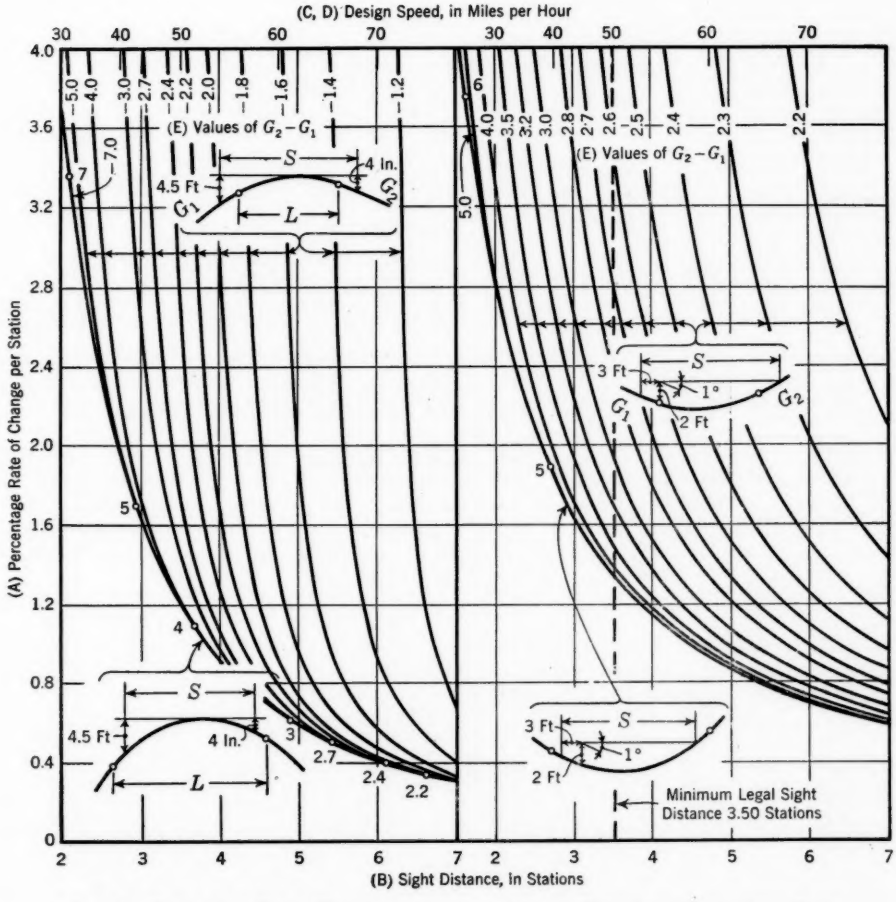


FIG. 8.—NONPASSING SIGHT DISTANCE OVER A SUMMIT

FIG. 9.—HEADLIGHT, NONPASSING SIGHT DISTANCE IN A SAG

value of $R = -1.0\%$ per station and $G_2 - G_1 = -4.5\%$, the interpolated value of S will be 5 ft greater than the calculated. An increase of 0.1 ft in the height of the driver's eye and the object above the road surface will increase the sight distance 74 ft at $R = -1.0\%$ per station and $G_2 - G_1 = -0.55\%$ and will increase the sight distance 9 ft at $R = -1.0\%$ per station and $G_2 - G_1 = -4.5\%$.

TABLE 3.—Sight Distances (in Stations) over

R	$S > L$	Values of 0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
													(a) Passing
	$S = \frac{6}{\sqrt{R}}$												$S < L$; S
−0.02	42.43	67.50	55.00	48.50	45.00	43.21	42.50	....	....	....	....	....	....
−0.03	34.64	65.00	51.67	44.30	40.00	37.38	35.83	35.00	34.67	....	....	....	....
−0.04	30.00	63.75	51.00	42.25	37.50	34.46	32.50	31.25	30.50	30.11	30.00	....	....
−0.06	24.49	62.50	48.33	40.17	35.00	31.54	29.16	27.50	26.33	25.53	25.00	24.68	24.52
−0.08	21.21	61.88	47.50	39.12	33.75	30.09	27.50	25.64	24.25	23.23	22.50	21.98	21.61
−0.10	18.97	61.50	47.00	38.50	33.00	29.21	26.50	24.50	23.00	21.86	21.00	20.35	19.86
−0.15	15.49	61.00	46.33	37.67	32.00	28.04	25.17	23.00	21.33	20.06	19.00	18.20	17.52
−0.20	13.42	60.75	46.00	37.25	31.50	27.46	24.50	22.25	20.50	19.14	18.00	17.10	16.36
−0.25	12.00	60.60	45.80	37.00	31.20	27.11	24.10	21.80	20.00	18.58	17.40	16.45	15.66
−0.30	10.95	60.50	45.67	36.83	31.00	26.88	23.83	21.50	19.66	18.21	17.00	16.02	15.19
−0.40	9.49	60.38	45.50	36.62	30.75	26.59	23.50	21.12	19.25	17.75	16.50	15.48	14.61
−0.50	8.49	60.30	45.40	36.50	30.60	26.41	23.30	20.90	19.00	17.47	16.20	15.15	14.26
−0.60	7.75	60.25	45.33	36.42	30.50	26.29	23.17	20.75	18.83	17.28	16.00	14.93	14.03
−0.80	6.71	60.19	45.25	36.31	30.38	26.15	23.00	20.56	18.62	17.06	15.75	14.66	13.73
−1.00	6.00	60.15	45.20	36.25	30.30	26.06	22.90	20.45	18.50	16.91	15.60	14.50	13.56
−1.20	5.48	60.12	45.17	36.21	30.25	26.00	22.83	20.38	18.42	16.82	15.50	14.39	13.44
−1.40	5.07	60.11	45.14	36.18	30.21	25.96	22.79	20.32	18.36	16.76	15.43	14.31	13.36
−1.60	4.74	60.09	45.12	36.16	30.19	25.93	22.75	20.28	18.31	16.71	15.37	14.26	13.30
−1.80	4.47	60.08	45.11	36.14	30.17	25.91	22.72	20.25	18.28	16.67	15.33	14.21	13.25
−2.00	4.24	60.07	45.10	36.12	30.15	25.88	22.70	20.22	18.25	16.64	15.30	14.18	13.20
−2.40	3.87	60.06	45.08	36.10	30.12	25.86	22.67	20.19	18.21	16.59	15.25	14.12	13.15
−3.00	3.46	60.05	45.07	36.08	30.10	25.83	22.63	20.15	18.17	16.54	15.20	14.07	13.09
−4.00	3.00	60.04	45.05	36.06	30.07	25.80	22.60	20.12	18.12	16.40	15.15	14.01	13.03
													(b) Nonpassing
	$S = \frac{3.8165}{\sqrt{R}}$												$S < L$; S
−0.02	26.99	31.94	28.33	27.16	....	....	....	....	....	....	....	....	....
−0.03	22.03	29.44	25.00	22.99	22.22	....	....	....	....	....	....	....	....
−0.04	19.08	28.19	23.33	20.91	19.72	19.32	....	....	....	....	....	....	....
−0.06	15.58	26.94	21.66	18.83	17.22	16.30	15.82	15.65	....	....	....	....	....
−0.08	13.49	26.32	20.83	17.79	15.97	14.85	14.16	13.77	13.58	....	....	....	....
−0.10	12.07	25.94	20.33	17.16	15.22	13.97	13.16	12.65	12.33	12.16	12.11	....	....
−0.15	9.85	25.44	19.66	16.33	14.22	12.80	11.83	11.15	10.66	10.33	10.11	10.07	9.90
−0.20	8.53	25.19	19.33	15.91	13.72	12.22	11.16	10.40	9.83	9.41	9.11	8.89	8.74
−0.25	7.63	25.04	19.13	15.66	13.42	11.87	10.76	9.95	9.33	8.86	8.51	8.24	8.04
−0.30	6.97	24.92	19.00	15.49	13.22	11.64	10.49	9.65	8.99	8.50	8.11	7.80	7.57
−0.40	6.03	24.82	18.83	15.28	12.97	11.35	10.16	9.27	8.58	8.04	7.61	7.26	6.99
−0.50	5.40	24.70	18.73	15.16	12.82	11.17	9.96	9.05	8.33	7.76	7.31	6.94	6.64
−0.60	4.93	24.69	18.66	15.08	12.72	11.05	9.83	8.90	8.16	7.58	7.11	6.72	6.41
−0.80	4.27	24.63	18.58	14.97	12.60	10.91	9.66	8.71	7.95	7.35	6.86	6.55	6.11
−1.00	3.82	24.59	18.53	14.91	12.52	10.82	9.56	8.60	7.83	7.21	6.71	6.29	5.94
−1.20	3.48	24.57	18.50	14.87	12.47	10.76	9.49	8.52	7.75	7.12	6.61	6.18	5.82
−1.40	3.23	24.55	18.47	14.84	12.43	10.72	9.45	8.47	7.69	7.06	6.54	6.10	5.74
−1.60	3.02	24.53	18.45	14.82	12.41	10.69	9.41	8.43	7.64	7.01	6.48	6.05	5.68
−1.80	2.84	24.52	18.44	14.80	12.39	10.67	9.38	8.40	7.61	6.97	6.43	6.00	5.63
−2.00	2.70	24.51	18.43	14.78	12.37	10.64	9.36	8.37	7.58	6.94	6.41	5.96	5.58
−2.40	2.46	24.50	18.41	14.76	12.34	10.62	9.33	8.33	7.54	6.89	6.36	5.91	5.53
−3.00	2.20	24.49	18.40	14.74	12.32	10.59	9.29	8.30	7.50	6.85	6.31	5.86	5.47
−4.00	1.91	24.48	18.38	14.72	12.30	10.56	9.26	8.26	7.45	6.80	6.26	5.80	5.41

A Vertical Curve When L is Not Equal to S

Change in Grade, $G_2 - G_1$:														R
1.5	1.6	1.8	2.0	2.2	2.4	2.7	3.0	3.4	4.0	5.0	7.0	10.0	12.0	
Sight Distance														
$= \frac{18}{G_2 - G_1} + \frac{G_2 - G_1}{2R}$														
....	....	....	....	....	....	....	....	....	....	....	....	....	....	−0.02
....	....	....	....	....	....	....	....	....	....	....	....	....	....	−0.03
....	....	....	....	....	....	....	....	....	....	....	....	....	....	−0.04
....	....	....	....	....	....	....	....	....	....	....	....	....	....	−0.06
21.38	21.25	....	....	....	....	....	....	....	....	....	....	....	....	−0.08
19.50	19.25	19.00	....	....	....	....	....	....	....	....	....	....	....	−0.10
17.00	16.58	16.00	15.66	15.51	....	....	....	....	....	....	....	....	....	−0.15
15.75	15.25	14.50	14.00	13.68	13.50	....	....	....	....	....	....	....	....	−0.20
15.00	14.45	13.60	13.00	12.58	12.30	12.07	12.00	....	....	....	....	....	....	−0.25
14.50	13.92	13.00	12.33	11.85	11.50	11.17	11.00	....	....	....	....	....	....	−0.30
13.87	13.25	12.25	11.50	10.93	10.50	10.05	9.75	9.55	....	....	....	....	....	−0.40
13.50	12.85	11.80	11.00	10.38	9.90	9.37	9.00	8.70	8.50	....	....	....	....	−0.50
13.25	12.58	11.50	10.66	10.01	9.50	8.92	8.50	8.13	7.83	....	....	....	....	−0.60
12.94	12.25	11.12	10.25	9.56	9.00	8.36	7.83	7.42	7.00	6.72	....	....	....	−0.80
12.75	12.05	10.90	10.00	9.28	8.70	8.02	7.50	7.00	6.50	6.10	....	....	....	−1.00
12.62	11.92	10.75	9.83	9.10	8.50	7.79	7.25	6.72	6.17	5.68	....	....	....	−1.20
12.53	11.82	10.64	9.71	8.97	8.36	7.63	7.07	6.51	5.93	5.39	5.07	....	....	−1.40
12.47	11.75	10.56	9.63	8.87	8.26	7.51	6.94	6.36	5.75	5.16	4.76	....	....	−1.60
12.42	11.69	10.50	9.56	8.79	8.17	7.42	6.83	6.24	5.61	4.92	4.51	....	....	−1.80
12.37	11.65	10.45	9.50	8.73	8.10	7.33	6.75	6.15	5.50	4.85	4.32	....	....	−2.00
12.31	11.58	10.38	9.42	8.64	8.00	7.23	6.63	6.01	5.33	4.64	4.03	....	....	−2.40
12.25	11.52	10.30	9.33	8.55	7.90	7.12	6.50	5.87	5.17	4.43	3.73	3.47	....	−3.00
12.19	11.45	10.22	9.25	8.45	7.80	7.00	6.38	5.73	5.00	4.22	3.44	3.05	3.00	−4.00
Distance														
$= \frac{7.331}{G_2 - G_1} + \frac{G_2 - G_1}{2R}$														
....	....	....	....	....	....	....	....	....	....	....	....	....	....	−0.02
....	....	....	....	....	....	....	....	....	....	....	....	....	....	−0.03
....	....	....	....	....	....	....	....	....	....	....	....	....	....	−0.04
....	....	....	....	....	....	....	....	....	....	....	....	....	....	−0.06
....	....	....	....	....	....	....	....	....	....	....	....	....	....	−0.08
....	....	....	....	....	....	....	....	....	....	....	....	....	....	−0.10
....	....	....	....	....	....	....	....	....	....	....	....	....	....	−0.15
8.64	8.58	....	....	....	....	....	....	....	....	....	....	....	....	−0.20
7.89	7.78	7.67	....	....	....	....	....	....	....	....	....	....	....	−0.25
7.39	7.25	7.07	6.99	....	....	....	....	....	....	....	....	....	....	−0.30
6.76	6.58	6.32	6.16	6.08	6.05	....	....	....	....	....	....	....	....	−0.40
6.39	6.18	5.87	5.66	5.53	5.45	5.42	....	....	....	....	....	....	....	−0.50
6.14	5.91	5.57	5.32	5.16	5.05	4.97	....	....	....	....	....	....	....	−0.60
5.82	5.58	5.19	4.91	4.71	4.55	4.41	4.33	4.28	....	....	....	....	....	−0.80
5.64	5.38	4.97	4.66	4.43	4.25	4.07	3.94	3.86	....	....	....	....	....	−1.00
5.51	5.25	4.82	4.49	4.25	4.05	3.84	3.69	3.57	3.51	....	....	....	....	−1.20
5.42	5.15	4.71	4.37	4.12	3.91	3.68	3.52	3.37	3.27	....	....	....	....	−1.40
5.36	5.08	4.63	4.29	4.02	3.81	3.56	3.38	3.22	3.09	....	....	....	....	−1.60
5.30	5.02	4.57	4.22	3.94	3.72	3.47	3.28	3.10	2.95	2.79	....	....	....	−1.80
5.26	4.98	4.52	4.16	3.88	3.65	3.39	3.19	3.01	2.84	2.72	....	....	....	−2.00
5.20	4.91	4.45	4.08	3.79	3.55	3.28	3.07	2.86	2.67	2.51	....	....	....	−2.40
5.14	4.85	4.37	3.99	3.70	3.45	3.17	2.94	2.72	2.53	2.30	....	....	....	−3.00
5.07	4.78	4.29	3.86	3.60	3.35	3.05	2.82	2.58	2.34	2.09	1.92	....	....	−4.00

Headlight Sight Distance in a Sag

The distance at which the headlights of a car will illuminate the road surface, may be a determining factor in establishing the sharpness of a vertical curve in a sag (see Fig. 9). It is not within the province of this paper to discuss the amount of illumination necessary to make the road surface visible to the

TABLE 4.—Sight Distances on a

Symbol[a]	Values of Change											
	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
	(a) Passing											
R	−0.0025	−0.0004	−0.0007	−0.001	−0.014	−0.018	−0.024	−0.028	−0.033	−0.040	−0.047	−0.054
S	120.00	90.00	72.00	60.00	51.40	45.00	40.00	36.00	32.73	30.00	27.69	25.71
	(b) Nonpassing											
R	−0.005	−0.01	−0.02	−0.025	−0.03	−0.04	−0.055	−0.07	−0.083	−0.10	−0.115	−0.13
S	48.87	36.66	29.30	24.20	20.94	18.30	16.29	14.66	13.33	12.20	11.28	10.46

[a] *R* = Percentage rate of change of grade per station;

driver. The Vehicle Code of the State of California specifies certain limitations on automobile headlight equipment, as follows[3]:

> "Headlamps shall be located at a height measured from the center of the headlamps of not more than fifty-four inches nor less than twenty-four inches above the level surface upon which said vehicle stands."

Obviously, the lesser height will provide illumination at a shorter distance. Hence a height of 2 ft has been used as a condition for the following calculations. Referring to multiple beam road lighting equipment, the code states:

> "There shall be an uppermost distribution of light, or composite beam, so aimed and of such intensity as to reveal persons and vehicles at a distance of at least 350 feet ahead for all conditions of loading. The maximum intensity of this uppermost distribution of light or composite beam one degree of arc or more above the horizontal level of the lamps when the vehicle is not loaded shall not exceed 8,000 apparent candlepower, and at no other point of the distribution of light or composite beam shall there be an intensity of more than 75,000 apparent candlepower."[4]

An analysis of this paragraph reveals that 350 ft is established as a minimum legal sight distance and that +1° is the maximum vertical angle of the effective light beam.

Distances are measured horizontally and vertically. Hence, in order to determine the true length of the sight distance in the simplest manner, the

[3] "Vehicle Code, State of California, as Amended to 1949," issued by Dept. of Motor Vehicles, Div. 10, Equipment, Chapter 2 (Required Lighting Equipment). Sect. 619, p. 253.

[4] *Ibid.*, Div. 10, Equipment, Chapter 4 (Test and Approval of Lamps, Lamp Equipment and Signal Devices), Sect. 648, Para. (a), p. 268.

light beam will be pictured as a horizontal line. The vertical angle between the road surface and the light beam is 1°, so the angle of the road surface will be 1° downward from the horizontal. The tangent of a 1° angle is 0.017455, defining a slope of 0.017455 ft per ft, or 1.7455 ft in 100 ft, which, in other conventional terms, is a grade of $-$ 1.7455%.

VERTICAL CURVE WHEN L IS EQUAL TO S

IN GRADE, $G_2 - G_1$:														Sym-bol[a]
1.5	1.6	1.8	2.0	2.2	2.4	2.7	3.0	3.4	4.0	5.0	7.0	10.0	12.0	
SIGHT DISTANCE, $R = \frac{(G_2 - G_1)^2}{36}$; $S = \frac{36}{G_2 - G_1}$														
−0.062 24.00	−0.07 22.50	−0.090 20.00	−0.11 18.00	−0.13 16.36	−0.16 15.00	−0.20 13.33	−0.25 12.00	−0.32 10.59	−0.44 9.00	−0.70 7.20	−1.36 5.14	−2.78 3.60	−4.00 3.00	R S
SIGHT DISTANCE, $R = \frac{(G_2 - G_1)^2}{14.662}$; $S = \frac{14.662}{G_2 - G_1}$														
−0.153 9.77	−0.18 9.15	−0.22 8.14	−0.27 7.32	−0.33 6.66	−0.39 6.10	−0.50 5.42	−0.61 4.88	−0.788 4.31	−1.09 3.66	−1.70 2.93	−3.37 2.09	....	....	R S

S = Sight distance, in hundreds of feet.

The headlamp is not less than 2 ft normal to the road surface (see Fig. 10), which height, on a $-$ 1.7455% grade, amounts to a vertical distance of 2.000304 ft. Such accuracy is not needed in this problem, and the vertical distance from the road surface to the light will be assumed as 2 ft.

The driver of a motor vehicle sits behind the headlamps an average distance of about 5 ft in most passenger cars. In trucks and buses that have the engine mounted under the seat or at the back, this distance may be as short as 2.5 ft. For purposes of calculating sight distance, a value of 3 ft will be added to the length of the light beam. Two general cases will be considered.

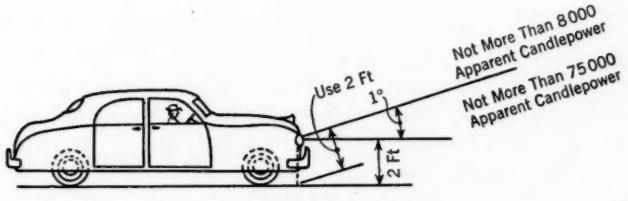


FIG. 10

Case 1.—Consider an indefinitely long curve—one in which the length of the curve is greater than the length of the light beam (see Fig. 11(*a*)). Designate the length of the light beam as B and the length of the part of the curve as L. The difference in elevation H between the two points on the curve is 2 ft.

The grade at the first point is $G_1 = -1.7455\%$. Eqs. 1 and 3 combined as—

$$L = \sqrt{\left(\frac{G_1}{R}\right)^2 + \frac{2H}{R}} - \frac{G_1}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

—fits this condition. Substituting the foregoing values, Eq. 30 becomes

$$B = \sqrt{\left(\frac{-1.7455}{R}\right)^2 + \frac{2\,(2)}{R}} - \frac{-1.7455}{R}$$

$$= \frac{1}{R}\left(\sqrt{3.0468 + 4R} + 1.7455\right) \ldots (31)$$

—and the sight distance will be $B + 0.03$ stations, or

$$S = \frac{1}{R}\left(\sqrt{3.0468 + 4R} + 1.7455\right) + 0.03 \quad \ldots\ldots\ldots\ldots (32)$$

—from which the sight distance for any desired value of rate of change may be determined.

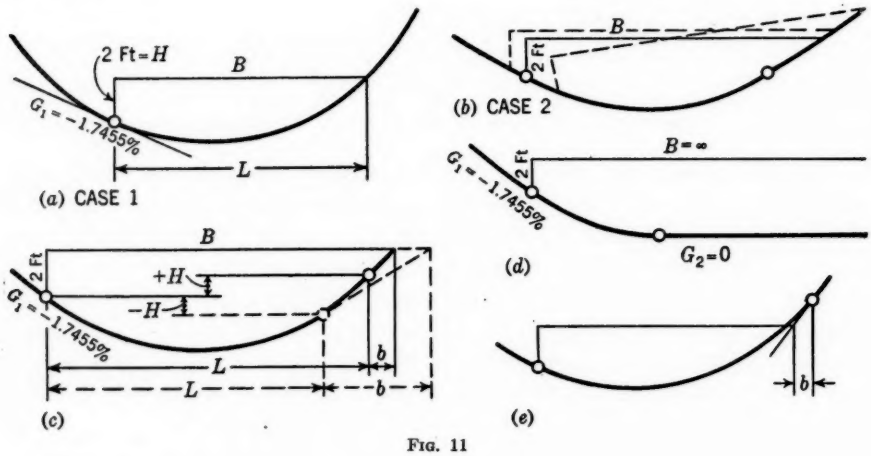


FIG. 11

Case 2.—Consider a vertical curve that is shorter than the light beam, as shown in Fig. 11(*b*). The shortest length of beam B will occur when the vehicle is at the beginning of the curve. Movement either way from this point will increase the length of B. Under this condition, B is composed of two parts L and b, in Fig. 11(*c*), distance $L = (G_2 - G_1)/R$ being the total length of the vertical curve and b being the length of the tangent between the end of the curve and the point of intersection with the light beam. Height H is now the difference in elevation of the two ends of the curve. By inspection

$$b = \frac{2 - H}{G_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

The elevation difference H may be positive or negative within certain limits. In the special case when H is 0, grade G_2 will be equal to $-G_1$ and b will be

2/+ 1.74455. Substituting Eq. 3 in Eq. 33,

$$b = \frac{2 - 0.5\,L\,(G_2 + G_1)}{G_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

In Eq. 34, substitute the value of L determined from Eq. 1, and substitute for $G_2 + G_1$ its equivalent $(G_2 - G_1) + 2\,G_1$—which is equal to $(G_2 + G_1) + 2\,(-1.7455)$ or $(G_2 - G_1) - 3.4910$. Also, substitute for G_2 its equivalent

TABLE 5.—HEADLIGHT, NONPASSING SIGHT DISTANCE IN A SAG (IN STATIONS), WHEN L IS NOT EQUAL TO S

R	S (Eq. 32, $S > L$)	VALUES OF $G_2 - G_1$:															
		2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.2	3.5	4.0	5.0	6.0
		$S < L$, S from Equation 36															
0.1	36.05	86.47	67.87	57.68	51.34	47.09	44.10	41.93	40.31	39.10	38.36	37.50	36.61	36.08			
0.2	18.56	47.18	36.77	31.05	27.49	25.09	23.39	22.15	21.22	20.51	20.06	19.56	19.01	18.64			
0.3	12.72	34.08	26.41	22.18	19.54	17.75	16.49	15.56	14.86	14.32	13.96	13.58	13.14	12.81			
0.4	9.78	27.54	21.22	17.74	15.56	14.09	13.04	12.26	11.67	11.22	10.91	10.59	10.21	9.90			
0.5	8.01	23.61	18.11	15.08	13.18	11.89	10.96	10.28	9.76	9.36	9.08	8.72	8.45	8.15			
0.6	6.83	20.99	16.04	13.31	11.59	10.42	9.58	8.96	8.49	8.12	7.86	7.54	7.27	6.99	6.83		
0.7	5.98	19.12	14.56	12.04	10.45	9.37	8.60	7.82	7.58	7.24	6.99	6.75	6.43	6.16	5.99		
0.8	5.34	17.71	13.45	11.09	9.61	8.59	7.86	7.32	6.90	6.57	6.35	6.11	5.81	5.53	5.35		
0.9	4.83	16.62	12.58	10.35	8.94	7.98	7.28	6.77	6.37	6.06	5.83	5.61	5.32	5.05	4.86		
1.0	4.43	15.75	11.89	9.76	8.41	7.49	6.82	6.33	5.94	5.64	5.42	5.21	4.93	4.67	4.47		
1.1	4.10	15.03	11.33	9.27	7.97	6.97	6.45	5.97	5.60	5.31	5.10	4.89	4.61	4.34	4.14		
1.2	3.82	14.44	10.86	8.87	7.61	6.75	6.13	5.67	5.31	5.02	4.81	4.61	4.34	4.08	3.87		
1.3	3.58	13.93	10.46	8.53	7.31	6.47	5.87	5.44	5.06	4.79	4.58	4.38	4.11	3.86	3.65		
1.4	3.38	13.50	10.12	8.23	7.04	6.23	5.64	5.20	4.85	4.58	4.38	4.19	3.92	3.66	3.44		
1.5	3.20	13.13	9.82	7.98	6.82	6.02	5.44	5.01	4.67	4.41	4.20	4.02	3.75	3.50	3.28		
1.6	3.04	12.80	9.56	7.76	6.62	5.84	5.27	4.84	4.51	4.25	4.05	3.87	3.61	3.35	3.14		
1.7	2.90	12.51	9.33	7.56	6.44	5.67	5.12	4.70	4.37	4.11	3.92	3.73	3.48	3.21	3.00		
1.8	2.78	12.25	9.13	7.39	6.29	5.53	4.98	4.57	4.25	3.98	3.80	3.62	3.36	3.11	2.89		
1.9	2.67	12.03	8.95	7.23	6.15	5.40	4.86	4.45	4.14	3.88	3.69	3.51	3.26	3.01	2.78	2.67	
2.0	2.56	11.82	8.78	7.09	6.02	5.29	4.75	4.35	4.04	3.79	3.59	3.42	3.17	2.92	2.69	2.57	
2.2	2.39	11.46	8.50	6.85	5.81	5.06	4.56	4.17	3.86	3.62	3.43	3.26	3.01	2.76	2.53	2.39	
2.4	2.24	11.16	8.26	6.65	5.62	4.92	4.41	4.02	3.72	3.48	3.29	3.12	2.87	2.62	2.40	2.25	
2.6	2.11	10.91	8.06	6.48	5.47	4.78	4.27	3.89	3.59	3.36	3.17	3.00	2.76	2.50	2.28	2.12	
2.8	2.00	10.70	7.89	6.33	5.34	4.66	4.16	3.78	3.49	3.25	3.08	2.91	2.66	2.42	2.19	2.02	
3.0	1.91	10.51	7.75	6.21	5.23	4.55	4.06	3.69	3.40	3.17	2.98	2.82	2.58	2.33	2.10	1.93	
3.2	1.82	10.34	7.62	6.10	5.13	4.46	3.98	3.61	3.32	3.09	2.91	2.75	2.51	2.26	2.03	1.85	
3.4	1.74	10.20	7.50	6.00	5.04	4.38	3.90	3.53	3.25	3.02	2.84	2.68	2.44	2.20	1.96	1.78	
3.6	1.69	10.07	7.40	5.91	4.96	4.31	3.83	3.47	3.19	2.96	2.78	2.62	2.38	2.14	1.90	1.71	
3.8	1.61	9.96	7.31	5.83	4.89	4.24	3.77	3.41	3.13	2.91	2.73	2.57	2.33	2.09	1.85	1.66	1.61
4.0	1.56	9.85	7.23	5.76	4.83	4.19	3.72	3.36	3.08	2.85	2.68	2.52	2.29	2.04	1.80	1.61	1.56

$(G_2 - G_1) + G_1$—which is equal to $(G_2 - G_1) + (-1.7455)$ or $(G_2 - G_1) - 1.7455$. Entering these several values in Eq. 34 and reducing,

$$b = \frac{4\,R - (G_2 - G_1)\,[(G_2 - G_1) - 3.491]}{2\,R\,[(G_2 - G_1) - 1.7455]} \quad \ldots\ldots\ldots\ldots (35)$$

From Fig. 11(*c*), $B = L + b$ and $S = B + 0.03 = L + b + 0.03$ stations. With proper substitutions,

$$S = \frac{G_2 - G_1}{R} + \frac{4\,R - (G_2 - G_1)\,[(G_2 - G_1) - 3.491]}{2\,R\,[(G_2 - G_1) - 1.7455]} + 0.03 \quad \ldots\ldots\ldots (36)$$

Examination of Eq. 36 reveals that, if $G_2 - G_1 = +1.7455$, in terms of Fig. 11(*d*), when $G_2 = 0$, the second term b will become infinite. Hence, the sight distance will be infinite regardless of the value of R. Also, when $H = 2$ (which is when the numerator of the second fraction in Eq. 36 equals zero), the second term, b, will be zero and the length of the curve is equal to B. In that case,

$$S = \frac{G_2 - G_1}{R} + 0.03 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

This condition will exist when

$$R = \frac{(G_2 - G_1)\,[(G_2 - G_1) - 3.491]}{4} \quad \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

—that is, when the numerator of the second fraction in Eq. 36 is zero. From Eq. 38, the values of R that will produce a curve of the same length as the light beam can be found for any desired values of $G_2 - G_1$. Conversely,

$$G_2 - G_1 = \sqrt{4R + 3.0468} + 1.7455 \quad \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

From Eq. 39 the value of $G_2 - G_1$ that will produce a curve of the same length as the light beam can be found for any desired values of R.

TABLE 6.—HEADLIGHT, NONPASSING SIGHT DISTANCE IN A SAG WHEN L IS EQUAL TO S

$G_2 - G_1$	R (Eq. 38)	S (Stations) (Eqs. 32 or 36)
3.4910	0.0000	Infinite
3.5	0.0079	403.46
4.0	0.5090	7.90
5.0	1.8862	2.68
6.0	3.7635	1.62
7.0	6.1408	1.17

For values of $G_2 - G_1$ equal to or greater than those determined by Eq. 39, the second term of Eq. 36 will be zero or negative, respectively. When negative, it indicates that b is measured backward to the stationing (Fig. 11(*e*)). If the expression $G_2 - G_1$ in Eq. 36 is replaced with its equivalent from Eq. 39, and reduced, it becomes identical with Eq. 32. Eqs. 32, 36, and 38 have been solved for desirable values of $G_2 - G_1$ and the results are presented in Tables 5 and 6 and Fig. 9.

The lower curve of Fig. 9 represents the relation between sight distance and rate of change when the curve is of greater length than the sight distance. When the difference of grades is such that the sight distance is greater than the length of the curve, the line labeled with the respective difference of grades applies.

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